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Grouping students according to their cognitive maps:

an action-research experienced in the changing background of Italian school system

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Les profondes innovations en cours dans le système scolaire italien imposent désormais à tous les enseignants de revoir leur idée de classe et leur rôle dans la classe. L'attention est portée sur les relations entre les élèves organisés en groupes et sur la façon dont ces relations favorisent les procédés d'apprentissage. La recherche-action conduite avec quelques enseignants d'écoles primaires, de collèges et de lycées a suggéré de mettre au point des procédures à proposer successivement sur une grande échelle dans le but de faire valoir les différents comportements des connaissances des étudiants comme une ressource et non comme une difficulté pour l'apprentissage de tout. Une question ouverte est soumise aux étudiants; ils peuvent y exprimer librement leur pensée dans les formes et dans le langage qu'ils estiment le plus approprié. L'enseignant construit à partir de chaque réponse une carte des connaissances qui décrit les associations faites par l'élève à partir du mot clé proposé. Avec cette activité, l'enseignant se rend compte de l'énorme variété de manières avec lesquelles les étudiants construisent et organisent leurs concepts.

The Italian school system is undergoing a moment of deep changes both at educational and administrative level: the most influential teachers and intellectuals have drawn up a document, which is now public and outlines the contents and forms of knowledge that should be mastered by all citizens regardless of their religion, social class, nationality or sex; the schools which once directly depended on the central system are now becoming autonomous and are allowed to plan their own curricula within a national frame of reference and to organise their own timetables, classes and forms; in secondary schools students will be asked to take an active part in the organisation of school activities; each school should now enrol between 500 and 900 students and will be provided with up-to-date multimedia equipment; a system of assessment of the quality of education is being developed: it will provide for schools to evaluate their results; the school system which once was formed by three levels: primary school (from 6 to 11), middle school (from 11 to 14) and secondary (from 14 to 19) will now turn into just two levels as in the other European countries. Education will be compulsory till the age of 16 and students will be more carefully directed in the choice of their future studies; universities for the first time are creating special courses for future teachers. The Italian school system is trying hard to conform its standards to the European ones, but these radical changes are somewhat puzzling to teachers and principals who are now freer from narrow administrative boundaries of past, but are also asked to take greater responsibilities. One of the most important problems to tackle has to do with the new standards of creating classes or groups. In the past, Italian headmasters grouped students/pupils by chronological order and tried to mix sexes, abilities, aptitudes and to meet wishes of the parents who tried to get the best teachers for their children. The new tendency of grouping students for learning purposes obviously goes against the old idea of a class formed by individuals who separately learn from the teacher. As Paul Cobb¹ put it “*student's mathematical interpretation, solutions, explanations, and justifications, are seen not only as individual acts, but*

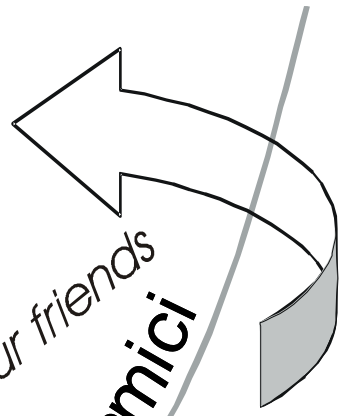
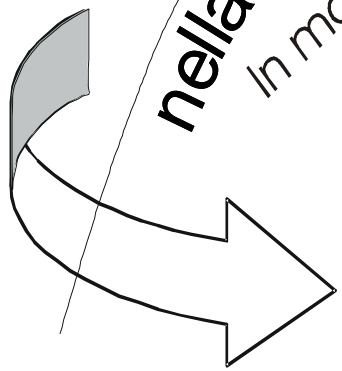
Figure 1 : the questionnaire

When you are talking to your friends
quando parli con gli amici

cos'è per te?

What does
.....
mean for you

nella matematica
In mathematics



simultaneously as acts of participating in collective or communal classroom processes. Viewed in this way mathematical learning is seen to be necessarily situated in a social context". This means that the creation of classes and groups must be in keeping with student's task and abilities, if we want to get to the desired processes of co-operative learning². The action research carried out with some teachers of primary and secondary schools aimed at testing some procedures which might be then proposed to a wider range of schools. Its aims are: to make teachers appreciate the different cognitive approaches of students as a resource and not as an obstacle to learning processes; to suggest a division into different working groups based on cognitive maps; to experiment with different ways of tackling problems with a class divided into different working groups; to make teachers appreciate the opportunities of cooperative learning. This experiences followed some stages, the same in all classes.

1) Handing out of the questionnaire with one question on the mathematics topic chosen.

The question is open-ended and a bit vague so as to let all the simple ideas, opinions, that some key words conjure up in the students, emerge clearly and freely. In order to compare the answers of students of different level the same frame has been used in all classes (figure 1). After a long discussion about the most important key word in the math curricula and about which of them should be posed in the classes of the researches, the following questions has been posed:

- "what does infinitesimal mean for you"* to 17 years old students
- "what does proportionality mean for you?"* to 14 years old students
- "what does a number mean for you?"* to 6 years old students

The reasons of such approach are exposed in the Cieaem 45 proceedings³

2) Processing each answer to derive a cognitive map that mirrors the students/pupils' schema of

knowledge The drawing of a map for each student has been easily accomplished by means of the analysis of their answers. In fact the teachers can draw cognitive maps which are of great use in the understanding of the way each student organises his learning processes. These maps are semantic structures which start from a concept that is considered the central point and to which all the following concepts are linked. Therefore they are a representation of the different organisation of concepts of each student, and the differences of sex, race, social class, interests and abilities are acknowledged and appreciated as the origin of their individual learning strategies and reaction (figure 2)

3) Grouping of students The action-research used cognitive maps⁴ as a standard by which groups are formed. By the analysis of the answers the teachers-researchers organised the classes in two different ways: homogeneous and heterogeneous groups in order to verify the hypothesis that open questions which allow a wide variety of different solutions are suitable with heterogeneous groups, while closed

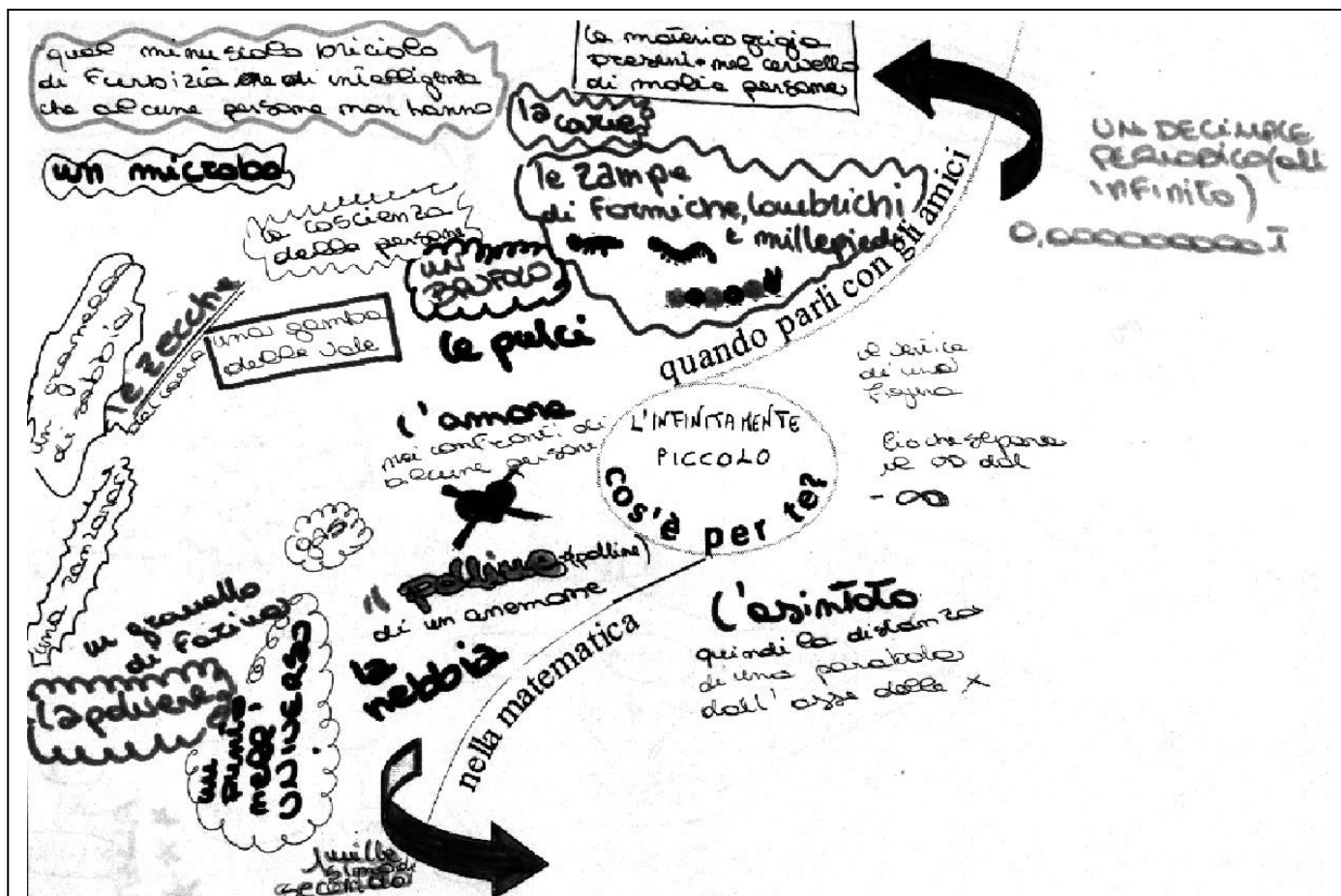
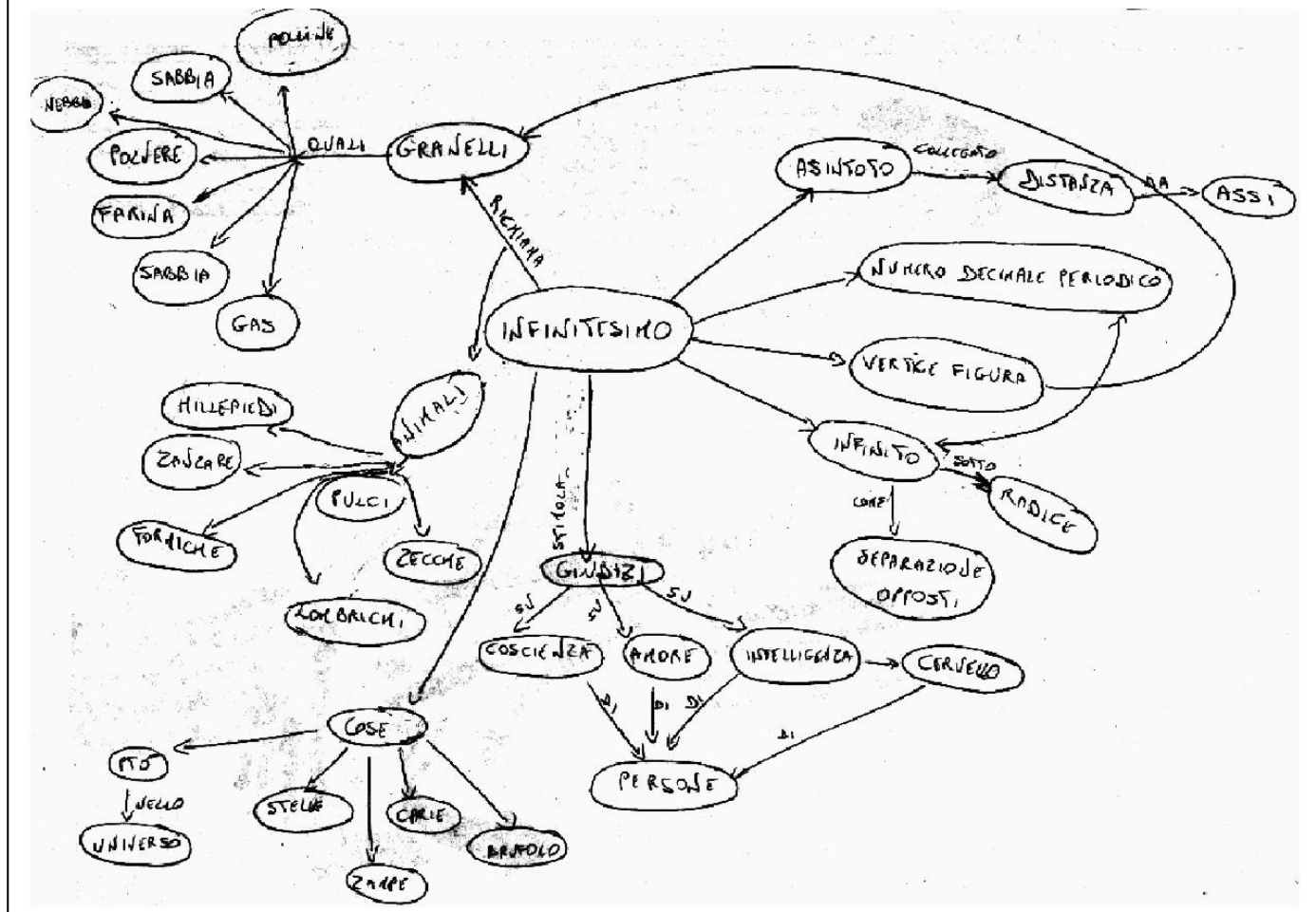


Figure 2: one answer and its map



problems which allow a narrow range of correct solution (or only one) are suitable with homogenous groups.

4) The teaching learning processes Let us describe in detail the teaching learning processes that follow such an approach.

Economics class of students aged 17, (4 male 17 female). The large majority of the students in the class connects the term “*infinitesimal*” with little things or animals (grains, pips, atoms, fleas, mosquitos,.....) many to a judgement of a human condition (metaphysical or personal) only some to the number zero and to a ratio of decreasing value. The teacher decides to involve the class in an investigation on infinite number sequence with the objective of pushing the large majority of the class beyond its static concepts. The students with the acquired knowledge of the definition of infinite number sequence are organised into heterogeneous groups which are involved in the following activities:

- to solve two problems adapted to a modern context from the Zenone paradox known as “Achilles and the turtle”. Trying to solve the problems the groups handle the concept of convergent sequences before having a lesson which explains them. With this initial experience the students are able to pose themselves the following question “can a sum of infinite number get a finite number?”
- to make research on internet starting from the key word “infinitesimal”
- preparing a poster resumming in a map their learnings from the previous activity.

Only now, after six hours of student investigation, the teacher, who up to now has suspended any judgement about the groups’ activities, holds a series of lessons on the formal definitions of limit of sequences. During the lesson the teacher methodically makes references to the poster designed by the students during the previous activity. In such a way the lesson is a sort of reorganisation of the concepts exposed by groups through the posters. After the lessons the teacher organises the classes into homogeneous groups giving them progressive difficulty problems applying the definition of limit of sequences to find the limit of some typical sequences.

Linguistic school of students aged 14 (3 male 26 female). Many students connect “*proportionality*” to the correct dimension of the human body, to the equilibrium in the relationship with friends, and to the formula of the equality of two ratios. The teacher with the objective of “meaning making, assigning meanings to things”⁵ in a mathematical setting decides to propose an investigation into the ratio between the height and the size of feet of each student organised into homogeneous groups. They have to fill in the following grid:

Name	Height (cm)	Size of your feet (cm)	Ratio
<i>one line for each student</i>	172	27	6,4

Then, starting from the grid each group has to write down on a poster with formulas, numbers, graphs, drawings, words or anything else that might express the “invariables” detected in the class. Starting from the results of posters and of the discussion carried out in class the teacher makes a lesson on the concept of proportionality and on the difference between proportions coming from demonstration, for instance in

geometry, and proportions coming from empirical measurement. After the lesson the teacher organises the class into homogeneous groups which are asked to find the subset of set “class 1BL” according to the ratio between height and foot size and to draw a representation using cartesian axes.

Elementary school of students aged 6 (11 female, 8 male one of them disabled). The large majority of pupils in their answers are proud (or worried) to be able to count. They are organised in heterogeneous groups in order to use Cousiner Gattegno⁶ rods to practise the concept of numbers, by doing a project. Of course they do not know the word “project” in fact they are asked to think before what to do and then to check if the result of their work is in keeping with the original thought. The groups work at different projects: some in two dimensions for instance landscape, others in three dimensions for instance a tower of decreasing section. A tv recording shows a typical process of cooperative learning: Fred a disabled child does not succeed in building the tower of decreasing section: the Cousiner Gattegno bars require a correct sequence of operations otherwise the tower falls down. Fred becomes nervous and anxious but Andrea, in his group, helps him. He shows him the correct sequence and Fred soon becomes autonomous in constructing towers. After such a success Fred feels himself strong enough to help somebody else in constructing towers!

4) Conclusions

At the end of the experiences the co-ordinator of the research interviews the teachers who noticed that: the work of the heterogeneous groups has been more interesting and stimulating than those of the homogeneous groups; many students appeared to have surprisingly different points of view. Some students who, during the school year, failed lots of subjects had taken an active role in the activities of the groups and produced very interesting answers to the questionnaire using a language of images and pictures. Therefore the teachers wondered if the use of more experiences of cooperative learning would have helped them to reach better results; in our schools it is possible and realistic to pay attention to forms of intelligence other than the logical and linguistic ones. In actual fact the cooperative learning can favour the success of pupils in difficulties if we use the right task for heterogeneous working groups. In such a context the teachers may appreciate or get the students to appreciate that the differences are just superficial fact if compared with the deep connections that link all men⁷.

References

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